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SATELLITE ORBITAL DATA

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ORBITAL INFORMATION

The orbital elements tabulated below have been derived by the indicated staff members of the Satellite Tracking Program, Smithsonian Astrophysical Observatory, employing the Differential Orbit Improvement Program (DOI) of G. Veis and C. Moore.

As opposed to osculating elements, the elements presented here are mean elements in the sense that the effects of the short period perturbations due to the earth's oblateness have been eliminated. These elements are called "SAO mean elements."

SAO mean elements have been derived from observations covering several days. The successive sets of elements are essentially independent of each other. They are dependent, however, in the sense that high-order coefficients in the secular and the long-periodic terms are generally considered as known and as constant for periods of several weeks or months, as dictated by convenience. Generally, these terms are sufficiently unimportant over several days that the orbits may be considered as truly independent.

The times of epoch in the mean elements are reckoned in Julian Days, but for the sake of convenience the number 2400000.5 has been subtracted in each case to provide an abbreviated notation which we call "Modified Julian Days," or "MJD."

Unless otherwise stated, the units of the orbital elements are degrees for angular quantities, megameters ($Mm = 10^6$ meters) for linear quantities, and revolutions for the mean anomaly M and its derivatives.

The tabulated values of the SAO mean elements give the values of arguments of perigee (ω), right ascension of the ascending node (Ω), inclination (i), eccentricity (e), and mean anomaly (M) as functions of time $t = T - T_0$ (where T_0 is the reference epoch) expressed in days.

The same tabulation also gives the mean (anomalistic) motion (n), the orbital acceleration ($n' = dn/dt$),¹ and the semimajor axis (a). The single digit placed at the right of each value represents the standard error for that element and refers to the last digit given. In the column M , we give the accumulated values of the mean anomaly at epoch but to avoid printing figures which are too big and unnecessary, we have subtracted systematically 25 revolutions every two days. Of the last three columns, the one headed N indicates the number of observations used for the computation of a set of elements; the one headed D , the number of days used; and the one headed σ , the mean standard error of the representation of the observations relative to their assumed accuracy.

SAO smoothed elements are derived either by a least-squares fit to the mean elements or directly by the DOI for the entire period they cover. These elements are used to obtain an approximate ephemeris when the accuracy of the mean elements is not required.

The elements are given as functions of time and they include in general both secular and periodic terms. The general expression for any element "b" is

¹Instead of n' , sometimes the value of $n'/2$ is tabulated

$$b = b_0 + b_1 t + b_2 t^2 + \dots + \sum A_i \sin(B_i + C_i t)$$

where $t = T - T_0$ is again expressed in days.

Since very often the effect of the third harmonic of the earth's potential is larger than the uncertainty of the elements, this effect may also be included in the form of $A \sin \omega$ or $B \cos \omega$.

The mean (anomalistic) motion n can be obtained from the smoothed elements by differentiating the expression for M , and the orbital acceleration $n' = dn/dt$ can be obtained by twice differentiating the same expression for M .

We repeat in part a note appearing in Special Report No. 28, page 7, on the orbital elements determined by the SAO Differential Orbit Improvement Program:

The reference plane is defined as the true equator of the date. The origin of right ascension is a line shifted from the mean equinox of the date by an amount equal to the precession in right ascension between 1950.0 and the date.

Given below is a formula with which values can be obtained to correct the right ascension given in the orbital elements, in a right ascension referring to the mean equinox of date:

$$\Omega^\circ(t) = \Omega^\circ(\text{DOI}) + 3^\circ.508 \times 10^{-5} (\text{MJD} - 33281)$$

where DOI indicates the values determined by the Differential Orbit Improvement Program, and distributed by the Smithsonian Astrophysical Observatory. MJD indicates the Modified Julian Day described above.

NOTICE

The following note refers to Table 1 on page 12 of SAO Special Report No. 50, dated October 3, 1960; and to the mean orbital elements published in SAO Special Report No. 40 (R) dated June 30, 1960 :

In the column M, we give the accumulated values of the mean anomaly at epoch, but to avoid printing figures which are too big and unnecessary, we have subtracted systematically 25 revolutions every two days.

ORBITAL ELEMENTS

For September, 1959, through April, 1960

Satellite 1958 B1 (Vanguard I Rocket)

Beatrice Miller

I. SAO smoothed elements

The following elements are based on 188 observations and are valid for the period September 1 through October 31, 1959.

$$\begin{aligned}
 T_0 &= 36841.0 \text{ MJD} \\
 \omega &= (295^\circ 869 \pm 13) + (4^\circ 1487 \pm 6)t + .735 \times 10^{-5}t^2 + .1083 \cos \omega \\
 \Omega &= (.532 \pm 6) - (2^\circ 8441 \pm 3)t - .503 \times 10^{-5}t^2 + .0133 \cos \omega \\
 i &= (34^\circ 270 \pm 2) - .70 \times 10^{-2} \sin \omega \\
 e &= (.20732 \pm 6) - .2627 \times 10^{-5}t + .4123 \times 10^{-3} \sin \omega \\
 M &= (.81114 \pm 2) + (10.400000 \pm 1)t + (.747 \pm 3) \times 10^{-5}t^2
 \end{aligned}$$

Standard error of one observation: $\sigma = \pm 4.8'$.

The following elements are based on 116 observations and are valid for the period November 1 through December 31, 1959.

$$\begin{aligned}
 T_0 &= 36902.0 \text{ MJD} \\
 \omega &= (189^\circ 120 \pm 15) + (4^\circ 1519 \pm 6)t + .735 \times 10^{-5}t^2 + .1083 \cos \omega \\
 \Omega &= (-172^\circ 919 \pm 6) - (2^\circ 8442 \pm 3)t - .503 \times 10^{-5}t^2 + .0133 \cos \omega \\
 i &= (34^\circ 272 \pm 2) - .70 \times 10^{-2} \sin \omega \\
 e &= (.20700 \pm 3) - .2627 \times 10^{-5}t + .4123 \times 10^{-3} \sin \omega \\
 M &= (.23476 \pm 3) + (10.400686 \pm 1)t + (.413 \pm 1) \times 10^{-5}t^2
 \end{aligned}$$

Standard error of one observation: $\sigma = \pm 4.0'$.

The following elements are based on 196 observations and are valid for the period January 1 through February 29, 1960.

$$\begin{aligned}
 T_0 &= 36964.0 \text{ MJD} \\
 \omega &= (86^\circ 446 \pm 21) + (4^\circ 1392 \pm 12)t + .735 \times 10^{-5}t^2 + .1083 \cos \omega \\
 \Omega &= (-349^\circ 158 \pm 7) - (2^\circ 8428 \pm 6)t - .503 \times 10^{-5}t^2 + .0133 \cos \omega \\
 i &= (34^\circ 275 \pm 2) - .70 \times 10^{-2} \sin \omega \\
 e &= (.20684 \pm 3) - .2627 \times 10^{-5}t + .4123 \times 10^{-3} \sin \omega \\
 M &= (.09466 \pm 4) + (10.401293 \pm 2)t + (.441 \pm 3) \times 10^{-5}t^2
 \end{aligned}$$

Standard error of one observation: $\sigma = \pm 6.0'$.

The following elements are based on 201 observations and are valid for the period March 1 through April 30, 1960.

$$\begin{aligned}
 T_0 &= 37024.0 \text{ MJD} \\
 \omega &= (-24^\circ 449 \pm 14) + (4^\circ 1567 \pm 9) t + .735 \times 10^{-5} t^2 + .1083 \cos \omega \\
 \Omega &= (200^\circ 203 \pm 4) - (2^\circ 8430 \pm 3) t - .503 \times 10^{-5} t^2 + .0133 \cos \Omega \\
 i &= (34^\circ 262 \pm 2) - .70 \times 10^{-2} \sin \omega \\
 e &= (.20678 \pm 1) - .2627 \times 10^{-5} t + .4123 \times 10^{-3} \sin \omega \\
 M &= (.18144 \pm 3) + (10.401585 \pm 2) t + (.344 \pm 1) \times 10^{-5} t^2
 \end{aligned}$$

Standard error of one observation: $\sigma = \pm 4.0'$.

II. SAO mean elements

CD	ω	Ω	i	e	M	n	$n^{1/2}$	a	N	D	σ
36812.0	175.37 4	82.97 3	34.28 1	.2069 2	.2182 1	10.39954 7	.16E-4 5	8.864666	19	8	1.11
36816.0	191.94 2	71.60 1	34.283 7	.2070 1	.81634 4	10.39953 1	.15E-4 2	8.864674	28	8	.90
36820.0	208.60 1	60.236 6	34.289 5	.20686 6	.41471 3	10.399651 9	.10E-4 2	8.864604	35	8	1.00
36824.0	225.23 2	48.877 9	34.292 6	.20678 7	.01346 3	10.39971 1	.4E-5 2	8.864573	21	8	.91
36828.0	241.87 9	37.50 6	34.28 1	.2065 2	.6125 1	10.39983 6	.2E-5 3	8.864501	18	8	3.73
36832.0	258.44 2	26.11 1	34.279 3	.20681 5	.21187 3	10.39985 1	.8E-5 6	8.86449	19	8	.84
36836.0	275.07 2	14.74 1	34.276 4	.20686 9	.81141 4	10.39993 1	.11E-4 2	8.864445	27	8	1.04
36840.0	291.72 2	3.378 7	34.279 2	.20677 7	.41124 2	10.40001 1	.6E-5 1	8.864401	38	8	.76
36844.0	308.36 2	352.007 8	34.277 2	.20671 7	.01133 3	10.400064 2	.89E-5 8	8.864368	36	8	.76
36848.0	324.96 3	340.643 9	34.273 3	.20683 8	.61172 5	10.400110 2	.5E-5 1	8.864342	33	8	.82
36852.0	341.61 4	329.27 2	34.266 4	.2070 1	.21220 8	10.400171 5	.0E-5 2	8.864307	43	8	1.09
36856.0	358.25 4	317.91 2	34.259 4	.2069 6	.8128 3	10.400204 4	.7E-5 1	8.864288	30	8	.63
36860.0	374.85 1	306.516 4	34.258 2	.20713 4	.41376 3	10.400274 2	.62E-5 6	8.864248	19	8	.45
36864.0	391.43 2	295.138 4	34.257 3	.20726 4	.01497 3	10.400328 3	.9E-5 1	8.864217	15	8	.47
36868.0	407.47 4	283.6 2	34.24 2	.197 6	.621 3	10.40052 8	.5E-4 3	8.864123	13	8	7.07
36874.0	72.8 1	-93.31 6	34.25 2	.2076 3	.0189 2	10.40048 8	.1E-4 1	8.86413	14	8	3.99
36878.0	89.4 1	-104.63 9	34.26 2	.2076 2	.6207 2	10.4004 1	.18E-4 8	8.864176	13	8	3.15
36882.0	105.97 3	-116.00 2	34.258 4	.20762 6	.22282 5	10.40056 2	.05E-4 2	8.864086	19	8	.79
36886.0	122.6 1	-127.4 1	34.27 4	.2073 6	.8248 4	10.40052 9	.02E-4 4	8.864111	17	8	1.14
36890.0	140. 1	-139.2 3	34.31 4	.208 2	.426 2	10.4004 1	.05E-4 3	8.86416	20	8	1.30
36894.0	156.1 4	-150.18 4	34.25 1	.2075 6	.0291 7	10.40059 7	.02E-4 1	8.86407	20	8	.83
36898.0	172.3 2	-161.58 5	34.28 2	.2070 3	.6322 4	10.4008 1	.20E-4 6	8.863946	14	8	1.71
36902.0	190.3 6	-173.1 2	34.21 8	.2059 6	.232 1	10.4000 6	.2E-4 2	8.864384	7	8	6.50
36904.0	197.5 3	-178.59 8	34.27 5	.2065 3	.0356 5	10.40070 1	.05E-4 4	8.864004	9	12	5.79
36908.0	213.5 6	-189.9 1	34.26 6	.208 1	.6397 9	10.4011 4	.1E-4 1	8.86376	11	12	12.98
36912.0	229.6 5	-201.3 1	34.26 6	.208 1	.2438 8	10.40075 6	.1E-4 1	8.863976	10	12	13.37
36916.0	246.0 5	-212.7 1	34.23 6	.209 1	.8469 7	10.40083 2	.3E-4 2	8.863927	11	12	12.14
36920.0	263.7 4	-224.1 1	34.28 7	.207 1	.4487 7	10.40085 3	.1E-4 1	8.863925	11	12	.89
36924.0	280.51 3	-235.493 7	34.288 2	.20647 8	.05180 4	10.400867 2	.047E-4 3	8.863913	22	12	.75
36928.0	297.12 1	-246.864 4	34.288 1	.20657 4	.65540 2	10.400905 1	.046E-4 3	8.863891	40	12	.49
36932.0	313.75 2	-258.238 5	34.288 1	.20664 5	.25910 3	10.400942 1	.047E-4 4	8.86387	54	12	.55

SAO mean elements (continued)

T (MJD)	ω	Ω	i	e	M	n	$n'/2$	a	K	D	σ
36934.0	322.04 2	-263.908 7	34.289 2	.20668 6	.06103 4	10.400975 5	.6E-5 1	8.863852	41 8	.53	
36938.0	338.62 3	-275.26 1	34.290 2	.20676 8	.66504 5	10.40104 3	.6E-5 1	8.863816	36 8	.60	
36942.0	355.16 3	-286.59 2	34.286 2	.2083 6	.2688 2	10.40105 4	.3E-5 1	8.86381	30 8	.53	
36946.0	371.67 6	-297.97 2	34.279 3	.2097 8	.8730 2	10.40117 9	.2E-5 2	8.863736	18 8	.51	
36950.0	388.6 3	-309.4 2	34.31 6	.208 4	.477 1	10.4016 5	.0E-4 1	8.863474	11 8	1.88	
36954.0	404.96 7	-320.75 2	34.29 1	.2073 2	.0826 2	10.40116 7	.2E-5 3	8.863746	16 8	1.35	
36958.0	421.57 2	-332.092 5	34.289 5	.20729 4	.68725 4	10.40119 1	.5E-5 1	8.863727	24 8	.80	
36962.0	438.12 2	-343.45 1	34.283 7	.20733 4	.29216 5	10.40128 2	.67E-5 8	8.863678	27 8	1.04	
36966.0	94.68 3	-354.83 1	34.274 5	.20737 4	.89729 5	10.401295 2	.40E-5 8	8.863668	26 8	1.05	
36970.0	111.29 2	-366.22 2	34.267 4	.20733 4	.50250 4	10.401340 2	.56E-5 6	8.863642	25 8	.80	
36974.0	127.88 4	-377.61 2	34.261 2	.20725 4	.10791 6	10.401371 2	.32E-5 8	8.863624	24 8	.73	
36978.0	144.44 3	-388.976 9	34.260 2	.20714 3	.71347 5	10.401379 2	.8E-5 7	8.863619	31 8	.56	
36982.0	160.99 4	-400.36 1	34.266 3	.20692 4	.31910 8	10.401414 2	.5E-5 1	8.86360	38 8	1.13	
36986.0	177.6 1	-411.71 3	34.25 1	.2068 1	.9248 2	10.401420 8	.5E-5 4	8.863596	39 8	3.55	
36990.0	194.28 3	-423.07 1	34.257 4	.20669 4	.53026 6	10.401437 3	.2E-5 1	8.863587	24 8	.96	
36994.0	210.96 4	285.54 1	34.255 5	.20656 4	.13592 7	10.401455 3	.02E-5 1	8.863577	21 8	1.05	
36998.0	227.57 4	274.17 1	34.255 6	.20652 4	.74178 7	10.401473 3	.3E-5 1	8.863566	20 8	.97	
37002.0	244.3 6	262.81 7	34.225 3	.206 3	.3474 6	10.40149 1	.10E-4 4	8.863555	14 8	2.40	
37006.0	261.2 7	251.38 5	34.226 4	.205 4	.9534 6	10.40151 2	.12E-4 6	8.863549	9 8	2.13	
37010.0	277.52 4	240.02 2	34.260 7	.20642 6	.55962 8	10.401522 4	.3E-5 2	8.863539	15 8	1.34	
37014.0	294.1 2	228.73 7	34.30 3	.2063 2	.1658 3	10.40155 1	.2E-5 9	8.863526	17 8	4.72	
37018.0	310.7 1	217.33 4	34.227 1	.2065 1	.7720 3	10.401554 9	.3E-5 4	8.863521	30 8	3.38	
37022.0	327.33 3	205.898 9	34.266 3	.20657 2	.37827 6	10.401580 2	.34E-5 8	8.863506	36 8	.90	
37026.0	344.00 3	194.52 1	34.264 3	.20668 2	.98457 6	10.401625 2	.2E-5 1	8.86348	42 8	.98	
37030.0	360.65 3	183.15 1	34.264 3	.20674 2	.59097 6	10.401647 2	.5E-5 1	8.863468	41 8	1.00	
37034.0	377.22 2	171.777 7	34.263 2	.20690 2	.19764 5	10.401677 2	.27E-5 7	8.863451	41 8	.75	
37038.0	393.80 3	160.401 8	34.263 2	.20700 2	.80439 5	10.401707 2	.4E-5 1	8.863433	32 8	.66	
37042.0	410.37 3	149.02 1	34.263 3	.20708 2	.41129 6	10.401733 2	.4E-5 1	8.863418	31 8	.74	
37046.0	427.00 3	137.65 1	34.260 4	.20717 3	.01812 7	10.401755 2	.17E-5 9	8.863406	37 8	.96	
37050.0	443.57 5	126.28 1	34.257 6	.20729 4	.6251 1	10.401780 4	.4E-5 2	8.863391	36 8	1.58	
37054.0	460.09 5	114.92 2	34.270 7	.20739 5	.2324 1	10.401802 4	.7E-5 2	8.863379	36 8	1.42	

TABLE 1

RELATIVE POSITIONS OF THE SUN AND THE PERIGEE OF SATELLITE 1958 81

T (MJD)	ω	Ω	Ψ	$\Delta\alpha$	φ	C (km)
36812.	175.37	82.97	99.13	99.65	2.60	0.0
36816.	191.88	71.63	99.10	98.38	-6.66	0.3
36820.	208.40	60.30	98.87	97.65	-15.54	1.5
36824.	224.93	48.96	99.13	98.15	-23.43	3.4
36828.	241.47	37.63	100.32	100.38	-29.65	5.3
36832.	258.02	26.29	102.59	104.40	-33.42	6.5
36836.	274.57	14.96	105.68	109.40	-34.15	6.8
36840.	291.12	3.63	109.09	113.99	-31.69	5.9
36844.	307.67	352.29	112.23	117.04	-26.47	4.3
36848.	324.22	340.96	114.55	118.23	-19.22	2.3
36852.	340.76	329.62	115.82	117.92	-10.70	0.7
36856.	357.29	318.28	116.20	116.75	-1.52	0.0
36860.	13.82	306.94	116.21	115.40	7.73	0.4
36864.	30.34	295.60	116.52	114.62	16.53	1.7
36868.	46.86	284.26	117.75	115.05	24.27	3.6
36872.	63.38	272.92	120.23	117.19	30.23	5.4
36876.	79.89	261.57	123.92	121.00	33.68	6.6
36880.	96.40	250.23	128.31	125.59	34.04	6.7
36884.	112.97	238.89	132.47	129.59	31.25	5.8
36888.	129.44	227.54	135.13	131.94	25.78	4.1
36892.	145.97	216.20	135.13	132.42	18.37	2.1
36896.	162.51	204.86	132.11	131.45	9.74	0.6
36900.	179.07	193.52	126.79	129.69	0.53	0.0
36904.	195.63	182.18	120.44	127.84	-8.73	0.5
36908.	212.20	170.84	114.34	126.64	-17.46	1.9
36912.	228.79	159.50	109.60	126.77	-25.06	3.9
36916.	245.38	148.17	107.00	128.69	-30.79	5.6
36920.	261.98	136.83	106.94	132.26	-33.89	6.7
36924.	278.58	125.49	109.35	136.51	-33.83	6.7
36928.	295.18	114.15	113.75	140.02	-30.64	5.6
36932.	311.78	102.81	119.35	141.84	-24.83	3.8
36936.	328.38	91.48	125.25	141.88	-17.17	1.9
36940.	344.97	80.13	130.56	140.60	-8.40	0.5
36944.	1.55	68.79	134.60	138.68	0.88	0.0
36948.	18.13	57.45	137.14	136.87	10.09	0.7
36952.	34.70	46.11	138.53	135.88	18.70	2.2
36956.	51.27	34.76	139.49	136.37	26.07	4.1
36960.	67.83	23.41	140.80	138.73	31.44	5.8
36964.	84.40	12.07	142.97	142.73	34.10	6.7
36968.	100.96	0.72	146.05	147.27	33.57	6.6
36972.	117.53	349.37	149.61	150.95	29.97	5.4
36976.	134.11	338.03	152.67	152.94	23.86	3.5
36980.	150.69	326.68	153.89	153.23	16.00	1.6
36984.	167.29	315.34	152.39	152.34	7.12	0.3
36988.	183.90	303.99	148.65	150.95	-2.19	0.0
36992.	200.51	292.65	144.01	149.78	-11.38	0.8
36996.	217.14	281.31	139.82	149.55	-19.88	2.5
37000.	233.78	269.97	137.11	150.90	-27.02	4.4
37004.	250.43	258.63	136.55	154.14	-32.04	6.0
37008.	267.08	247.29	138.50	158.87	-34.22	6.8
37012.	283.73	235.94	142.95	163.88	-33.16	6.4
37016.	300.38	224.60	149.56	167.78	-29.06	5.1
37020.	317.03	213.26	157.66	169.86	-22.57	3.2
37024.	333.68	201.92	165.84	170.23	-14.46	1.3
37028.	350.32	190.57	169.50	169.45	-5.43	0.2
37032.	6.95	179.23	163.88	168.22	3.91	0.1
37036.	23.58	167.88	154.95	167.28	13.02	1.1
37040.	40.20	156.53	146.26	167.34	21.32	2.8
37044.	56.82	145.18	139.02	168.99	28.13	4.8
37048.	73.43	133.83	133.96	172.45	32.68	6.3

I. SAO smoothed elements

The following elements are based on 377 observations and are valid for the period September 1 through October 31, 1959.

$$T_0 = 36841.0 \text{ MJD}$$

$$\omega = (83^\circ 373 \pm 6) + (4^\circ 4190 \pm 4) t + ?1922 \times 10^{-4} t^2 + ?1214 \cos \omega$$

$$\Omega = (-100^\circ 445 \pm 4) - (3^\circ 0224 \pm 2) t - ?1486 \times 10^{-4} t^2 + ?0123 \cos \omega$$

$$i = (34^\circ 255 \pm 1) - ?70 \times 10^{-2} \sin \omega$$

$$e = (.18972 \pm 2) - .1058 \times 10^{-5} t + .5698 \times 10^{-3} \sin \omega$$

$$M = (.54229 \pm 2) + (10.742195 \pm 1) t + (.459 \pm 2) \times 10^{-5} t^2$$

Standard error of one observation : $\sigma = \pm 4.5'$.

The following elements are based on 311 observations and are valid for the period November 1 through December 31, 1959.

$$T_0 = 36902.0 \text{ MJD}$$

$$\omega = (352^\circ 803 \pm 4) + (4^\circ 4136 \pm 3) t + ?1922 \times 10^{-4} t^2 + ?1214 \cos \omega$$

$$\Omega = (-284^\circ 839 \pm 2) - (3^\circ 0237 \pm 1) t - ?1486 \times 10^{-4} t^2 + ?0123 \cos \omega$$

$$i = (34^\circ 254 \pm 1) - ?70 \times 10^{-2} \sin \omega$$

$$e = (.18973 \pm 1) - .1058 \times 10^{-5} t + .5698 \times 10^{-3} \sin \omega$$

$$M = (.83021 \pm 1) + (10.742649 \pm 1) t + (.409 \pm 1) \times 10^{-5} t^2$$

Standard error of one observation : $\sigma = \pm 2.5'$.

The following elements are based on 314 observations and are valid for the period January 1 through February 29, 1960.

$$T_0 = 36964.0 \text{ MJD}$$

$$\omega = (626^\circ 548 \pm 4) + (4^\circ 4143 \pm 3) t + ?1922 \times 10^{-4} t^2 + ?1214 \cos \omega$$

$$\Omega = (-472^\circ 278 \pm 2) - (3^\circ 0235 \pm 1) t - ?1486 \times 10^{-4} t^2 + ?0123 \cos \omega$$

$$i = (34^\circ 247 \pm 1) - ?70 \times 10^{-2} \sin \omega$$

$$e = (.18981 \pm 1) - .1058 \times 10^{-5} t + .5698 \times 10^{-3} \sin \omega$$

$$M = (.88514 \pm 1) + (10.742967 \pm 1) t + .3000 \times 10^{-5} t^2$$

Standard error of one observation : $\sigma = \pm 2.5'$.

The following elements are based on 262 observations and are valid for the period
March 1 through April 22, 1960.

$$T_o = 37024.0 \text{ MJD}$$

$$\omega = (171^\circ 472 \pm 2) + (4^\circ 4168 \pm 2)t + 1922 \times 10^{-4}t^2 + 1214 \cos \omega$$

$$\Omega = (66^\circ 299 \pm 1) - (3^\circ 0244 \pm 1)t - 1486 \times 10^{-4}t^2 + 0123 \cos \Omega$$

$$i = (34^\circ 2445 \pm 4) - 70 \times 10^{-2} \sin \omega$$

$$e = (.189629 \pm 7) - .96 \times 10^{-5}t + .421 \times 10^{-3} \sin \omega$$

$$M = (.471574 \pm 7) + (10.743186 \pm 1)t + (.1286 \pm 9) \times 10^{-5}t^2$$

Standard error of one observation: $\sigma = \pm 1.0'$.

II. SAO mean elements

	ω	Ω	i	e	M	n	$n'/2$	a	N	D	σ
36814.0	324.221 9	-18.791 4	34.248 1	.18947 3	.50637 2	10.741885 2	.12E-4 1	8.675259	57	8	.54
36818.0	341.954 7	-30.910 4	34.258 1	.18952 2	.47402 2	10.741974 1	.86E-5 6	8.675212	64	8	.54
36822.0	359.631 6	-43.018 4	34.255 2	.18968 2	.44200 2	10.742010 1	.49E-5 8	8.675192	56	8	.59
36826.0	377.274 6	-55.113 4	34.252 2	.18992 2	.41016 2	10.742056 2	.17E-5 9	8.675167	44	8	.50
36830.0	394.937 7	-67.223 4	34.256 2	.19003 2	.37841 2	10.742092 2	.38E-5 7	8.675148	43	8	.38
36834.0	412.552 4	-79.300 3	34.237 1	.19024 1	.34691 1	10.742150 1	.115E-4 6	8.675115	40	8	.28
36838.0	430.14 1	-91.370 8	34.269 6	.19007 5	.31566 3	10.742219 3	.8E-5 2	8.675081	42	8	1.55
36842.0	447.805 6	-103.462 3	34.219 2	.19044 2	.28450 2	10.742242 1	.13E-5 8	8.675064	40	8	.35
36846.0	96.619 8	-109.513 3	34.222 2	.19037 2	.76898 2	10.742270 9	.79E-5 6	8.675049	46	8	.38
36848.0	114.234 7	-121.606 3	34.235 1	.19027 2	.73815 2	10.74237 1	.0E-6 7	8.674999	50	8	.38
36852.0	131.801 5	-133.691 3	34.239 1	.19016 2	.70753 1	10.742318 5	.21E-5 6	8.675025	47	8	.45
36856.0	149.44 1	-145.786 4	34.245 2	.18995 3	.67680 3	10.74235 1	.7E-5 1	8.675008	41	8	.51
36860.0	167.092 7	-157.884 3	34.253 1	.18974 2	.64620 2	10.742313 8	.21E-5 9	8.675029	45	8	.30
36864.0	184.813 8	-169.974 3	34.249 1	.18955 2	.61547 2	10.74236 1	.6E-5 1	8.675003	64	8	.31
36868.0	202.48 1	-182.056 4	34.248 1	.18946 2	.58502 2	10.74239 1	.37E-5 8	8.67499	52	8	.50
36874.0	228.97 2	-200.164 7	34.222 3	.18928 3	.03955 4	10.74244 2	.4E-5 2	8.674959	50	8	1.00
36878.0	246.65 2	-212.244 7	34.216 3	.18927 4	.00935 4	10.74248 2	.3E-5 2	8.674939	52	8	.94
36882.0	264.48 2	-224.359 6	34.235 4	.18937 4	.97899 4	10.74238 2	.3E-5 2	8.674992	38	8	.73
36886.0	282.16 2	-236.461 7	34.213 6	.18940 4	.94905 4	10.74251 2	.0E-5 2	8.674919	38	8	.85
36890.0	299.864 6	-248.563 4	34.264 2	.18945 2	.91918 2	10.742510 7	.0E-5 1	8.674924	43	8	.39
36894.0	317.578 6	-260.644 4	34.262 2	.18950 1	.88927 2	10.742566 7	.70E-5 8	8.674893	45	8	.36
36898.0	335.201 6	-272.713 5	34.262 2	.18957 1	.85976 2	10.742653 9	.19E-5 8	8.674846	55	8	.43
36902.0	352.86 1	-284.801 8	34.266 2	.18965 2	.83058 4	10.742638 3	.6E-5 2	8.674857	47	8	.70
36906.0	370.56 2	63.10 1	34.261 2	.18978 3	.80110 6	10.742691 4	.13E-4 2	8.674823	37	8	.65
36910.0	388.22 1	50.982 9	34.253 2	.18990 2	.77198 4	10.742718 3	.5E-5 1	8.674804	34	8	.46
36914.0	405.83 1	38.845 6	34.261 2	.19007 3	.74306 4	10.742710 4	.5E-5 2	8.674806	42	8	.64
36918.0	423.40 7	26.78 2	34.243 8	.1903 1	.7141 3	10.74289 2	.6E-4 1	8.674703	30	8	1.84
36922.0	441.062 9	14.685 2	34.244 1	.19002 2	.68519 3	10.742768 2	.67E-5 9	8.674769	32	8	.24
36926.0	458.72 1	2.583 2	34.249 1	.19004 3	.65616 3	10.742796 4	.8E-5 4	8.674755	18	8	.26
36930.0	476.39 1	-9.513 3	34.254 2	.19005 3	.62722 4	10.742787 7	.2E-5 4	8.674761	20	8	.33

SAO mean elements (continued)

T (MJD)	ω	Ω	i	e	M	n	$n'/2$	a	N	D	σ
36934.0	133.914 9	-21.604 2	34.250 1	.19003 2	.59894 3	10.74289 1	.1E-5 1	8.674718	38	8	.34
36938.0	151.59 1	-33.650 5	34.241 2	.18977 2	.56978 4	10.742823 4	.8E-5 2	8.674747	30	8	.41
36942.0	169.172 9	-45.783 5	34.244 1	.18958 2	.54148 3	10.742821 2	.2E-5 1	8.674752	35	8	.29
36946.0	186.87 1	-57.902 7	34.250 3	.18952 4	.51282 5	10.742874 3	.25E-4 3	8.674728	34	8	.48
36950.0	204.66 1	-70.007 6	34.254 3	.18961 3	.48407 3	10.742862 4	.0E-5 3	8.674739	27	8	.62
36954.0	222.332 6	-82.067 5	34.245 2	.18947 2	.45563 2	10.742882 3	.3E-5 2	8.674732	32	8	.52
36958.0	240.042 6	-94.132 5	34.247 2	.18949 2	.42719 2	10.742895 3	.9E-5 2	8.674727	51	8	.65
36962.0	257.717 7	-106.193 7	34.247 3	.18943 3	.39895 2	10.742929 3	.2E-5 2	8.674711	39	8	.48
36968.0	284.25 1	-124.334 5	34.261 3	.18942 3	.85680 3	10.74302 1	.6E-5 2	8.674647	28	8	.50
36972.0	301.910 9	-136.434 4	34.261 2	.18944 4	.82896 2	10.74307 1	.4E-5 1	8.67462	31	8	.41
36976.0	319.590 7	-148.549 3	34.259 1	.18952 2	.80119 2	10.74304 1	.1E-5 1	8.674639	35	8	.33
36980.0	337.281 6	-160.659 2	34.250 1	.18953 1	.77342 2	10.743082 9	.39E-5 8	8.674615	55	8	.33
36984.0	354.939 6	-172.757 2	34.247 1	.18961 1	.74578 1	10.743081 5	.24E-5 4	8.674615	56	8	.27
36988.0	372.595 5	-184.844 2	34.243 1	.18973 1	.71817 1	10.743099 5	.24E-5 5	8.674605	53	8	.23
36992.0	390.232 5	-196.924 5	34.235 1	.18984 1	.69063 1	10.743108 6	.38E-5 7	8.67460	56	8	.28
36994.0	39.059 9	157.018 4	34.231 1	.18984 2	.17687 2	10.743146 2	.6E-5 1	8.674579	36	8	.47
36998.0	56.714 7	144.934 3	34.231 1	.19005 2	.14937 2	10.743151 2	.4E-5 2	8.674576	31	8	.28
37002.0	74.329 6	132.841 3	34.230 2	.19020 2	.12198 2	10.743178 2	.13E-5 1	8.674561	31	8	.26
37006.0	91.974 3	120.734 1	34.234 1	.19027 1	.09464 1	10.743177 1	.06E-6 3	8.674562	37	8	.17
37010.0	109.569 8	108.648 3	34.246 2	.19009 2	.06745 2	10.743185 1	.19E-5 9	8.674558	48	8	.30
37014.0	126.98 2	96.588 7	34.228 4	.18970 4	.06086 7	10.743196 3	.12E-4 2	8.674552	45	8	.53
37018.0	144.844 5	84.409 3	34.244 2	.18995 1	.01289 2	10.743194 1	.53E-5 7	8.674554	44	8	.26
37022.0	162.512 5	72.299 3	34.238 2	.18989 2	.98557 2	10.743197 1	.35E-5 5	8.674552	51	8	.31
37028.0	189.067 6	54.165 3	34.246 1	.18968 2	.44451 2	10.743211 2	.11E-4 1	8.674545	37	8	.28
37032.0	206.734 5	42.094 3	34.252 1	.18958 2	.41733 2	10.743206 1	.8E-6 9	8.674548	41	8	.23
37036.0	224.384 5	30.007 3	34.255 1	.18939 2	.39020 2	10.743214 2	.6E-5 1	8.674544	27	8	.20
37040.0	242.076 5	17.920 2	34.261 1	.18935 2	.36301 1	10.743220 2	.1E-5 1	8.674541	27	8	.18
37044.0	259.800 9	5.802 2	34.260 1	.18934 2	.33584 2	10.743234 4	.8E-5 3	8.674534	31	8	.25

TABLE 2
RELATIVE POSITIONS OF THE SUN AND THE PERIGEE OF SATELLITE 1958 B2

T (MJD)	ω	Ω	Ψ	$\Delta\alpha$	φ	C (km)
36814.	324.22	172.00	33.58	339.91	-19.21	2.3
36818.	341.00	159.00	25.98	339.86	-10.07	0.7
36822.	359.57	147.80	21.68	338.92	-0.24	0.0
36826.	17.22	135.70	22.74	337.96	9.59	0.6
36830.	34.87	123.60	27.51	337.85	18.76	2.2
36834.	52.51	111.50	32.89	339.35	26.51	4.3
36838.	70.14	99.39	36.94	342.91	31.95	5.0
36842.	87.77	87.29	38.67	348.10	34.21	6.8
36846.	105.40	75.18	37.72	353.49	32.85	6.3
36850.	123.03	63.08	34.12	357.49	28.15	4.8
36854.	140.67	50.97	28.31	359.41	20.89	2.7
36858.	158.32	38.87	20.90	359.51	12.00	0.9
36862.	175.97	26.76	12.70	358.53	2.26	0.0
36866.	193.65	14.66	4.91	357.31	-7.63	0.4
36870.	211.33	2.56	5.01	356.73	-17.01	1.8
36874.	229.02	350.46	10.94	357.60	-25.14	3.9
36878.	246.73	338.35	15.45	0.48	-31.13	5.7
36882.	264.43	326.25	17.80	5.15	-34.07	6.7
36886.	282.14	314.15	17.96	10.31	-33.38.	6.5
36890.	299.85	302.05	16.53	14.27	-29.22	5.1
36894.	317.55	289.94	15.23	16.14	-22.32	3.1
36898.	335.24	277.84	16.88	16.04	-13.63	1.2
36902.	352.93	265.73	22.58	14.67	-3.97	0.1
36906.	10.60	253.62	30.76	12.90	5.94	0.2
36910.	28.27	241.51	39.73	11.60	15.45	1.5
36914.	45.92	229.40	48.19	11.62	23.84	3.5
36918.	63.57	217.28	5.509	13.58	30.25	5.5
36922.	81.21	205.17	59.56	17.46	33.78	6.6
36926.	98.85	193.05	60.98	22.17	33.77	6.6
36930.	116.49	180.94	59.11	26.03	30.24	5.4
36934.	134.14	168.82	54.21	27.98	23.82	3.5
36938.	151.80	156.71	46.97	27.97	15.42	1.5
36942.	169.46	144.60	38.39	26.64	-5.91	0.2
36946.	187.14	132.48	29.87	24.83	-4.01	0.1
36950.	204.83	120.37	23.50	23.43	-13.67	1.2
36954.	222.54	108.26	21.80	23.30	-22.36	3.1
36958.	240.25	96.14	24.93	25.13	-29.25	5.1
36962.	257.97	84.03	30.07	29.06	-33.40	6.5
36966.	275.69	71.92	34.90	34.17	-34.06	6.7
36970.	293.41	59.81	38.38	38.78	-31.10	5.7
36974.	311.12	47.69	40.28	41.60	-25.08	3.9
36978.	328.83	35.58	41.00	42.41	-16.93	1.8
36982.	346.54	23.46	41.36	41.76	-7.53	0.4
36986.	4.23	11.35	42.33	40.48	2.38	0.0
36990.	21.91	359.23	44.61	39.45	12.12	0.9
36994.	39.58	347.11	48.25	39.52	21.01	2.8
36998.	57.24	334.98	52.67	41.42	28.24	4.8
37002.	74.99	322.86	57.02	45.41	32.90	6.3
37006.	92.55	310.73	60.46	50.76	34.20	6.8
37010.	110.20	298.61	62.42	55.89	31.87	6.0
37014.	127.86	286.49	62.76	59.36	26.37	4.2
37018.	145.52	274.36	61.82	60.78	18.57	2.2
37022.	163.20	262.24	60.45	60.58	9.36	0.6
37026.	180.89	250.11	59.74	59.57	-0.50	0.0
37030.	198.59	237.99	60.69	58.60	-10.33	0.7
37034.	216.30	225.87	63.78	58.54	-19.46	2.4
37038.	234.02	213.75	68.71	60.18	-27.09	4.5
37042.	251.75	201.62	74.60	63.90	-32.31	6.1

Satellite 1958 52 (Sputnik III)

Y. Kozai

SAO smoothed elements. (An underline indicates an assumed value.)

The following elements are based on 138 observations and are valid for the period September 1.0 through September 9.0, 1959.

$$T_o = 36816.0 \text{ MJD}$$

$$\omega = \underline{233^{\circ}4215} - \underline{.41122 t} - \underline{1^{\circ}212 \times 10^{-4} t^2}$$

$$\Omega = (137^{\circ}913 \pm 12) - (3^{\circ}03098 \pm 95) t - \underline{8^{\circ}065 \times 10^{-4} t^2}$$

$$i = 65^{\circ}1057 \pm 71$$

$$e = (.06525 \pm 27) + (2.64 \pm 91) \times 10^{-4} t - \underline{5.2 \times 10^{-6} t^2}$$

$$M = (.111400 \pm 84) + (14.762027 \pm 28) t + (1.7527 \pm 13) \times 10^{-3} t^2 - (2.104 \pm 54) \times 10^{-5} t^3$$

Standard error of one observation: $\sigma = \pm 7.5'$.

The following elements are based on 43 observations and are valid for the period September 9.0 through September 17.0, 1959.

$$T_o = 36824.0 \text{ MJD}$$

$$\omega = \underline{230^{\circ}1240} - \underline{.41316 t} - \underline{1^{\circ}212 \times 10^{-4} t^2}$$

$$\Omega = (113^{\circ}6752 \pm 89) - (3^{\circ}0318 \pm 18) t - \underline{8^{\circ}065 \times 10^{-4} t^2}$$

$$i = 65^{\circ}139 \pm 10$$

$$e = (.06233 \pm 24) - (.03 \pm 11) \times 10^{-3} t - \underline{5.2 \times 10^{-7} t^2}$$

$$M = (.308821 \pm 78) + (14.787285 \pm 32) t + (1.5886 \pm 25) \times 10^{-3} t^2 + (.74 \pm 64) \times 10^{-6} t^3$$

Standard error of one observation: $\sigma = \pm 4.5'$.

The following elements are based on 31 observations and are valid for the period September 17.0 through September 25.0, 1959.

$$T_o = 36832.0 \text{ MJD}$$

$$\omega = (226^{\circ}811 \pm 82) - \underline{.415098 t} - \underline{1^{\circ}212 \times 10^{-4} t^2}$$

$$\Omega = (89^{\circ}380 \pm 16) - (3^{\circ}0503 \pm 33) t - \underline{8^{\circ}065 \times 10^{-4} t^2}$$

$$i = 65^{\circ}148 \pm 22$$

$$e = (.06211 \pm 13) - (1.94 \pm 34) \times 10^{-4} t - \underline{5.2 \times 10^{-7} t^2}$$

$$M = (.71228 \pm 20) + (14.814596 \pm 20) t + (1.8976 \pm 41) \times 10^{-3} t^2 + (.63 \pm 17) \times 10^{-5} t^3$$

Standard error of one observation: $\sigma = \pm 7.5'$.

The following elements are based on 73 observations and are valid for the period September 25.0 through October 3.0, 1959.

$$T_o = 36840.0 \text{ MJD}$$

$$\omega = (223^\circ 40 \pm 15) - \frac{41704}{\text{t}} - \frac{1^\circ 212 \times 10^{-4} t^2}{\text{t}^2}$$

$$\Omega = (64^\circ 9682 \pm 66) - (3^\circ 0549 \pm 24) \text{t} - \frac{8^\circ 065 \times 10^{-4} t^2}{\text{t}^2}$$

$$i = 65^\circ 153 \pm 10$$

$$e = (.060610 \pm 79) - (2.62 \pm 39) \times 10^{-4} \text{t} - \frac{5.2 \times 10^{-7} t^2}{\text{t}^2}$$

$$M = (.35236 \pm 47) + (14.845352 \pm 9) \text{t} + (1.8916 \pm 51) \times 10^{-3} \text{t}^2 - (.18 \pm 10) \times 10^{-5} \text{t}^3$$

Standard error of one observation: $\sigma = \pm 6.0'$.

The following elements are based on 102 observations and are valid for the period October 3.0 through October 13.0, 1959.

$$T_o = 36846.0 \text{ MJD}$$

$$\omega = (220^\circ 29 \pm 21) - \frac{4175}{\text{t}} - \frac{1^\circ 0 \times 10^{-4} t^2}{\text{t}^2}$$

$$\Omega = (46.583 \pm 11) - (3.0704 \pm 16) \text{t} - \frac{1^\circ 128 \times 10^{-4} t^2}{\text{t}^2}$$

$$i = 65.1378 \pm 56$$

$$e = (.05984 \pm 10) - (1.05 \pm 17) \times 10^{-4} \text{t} - \frac{1.29 \times 10^{-7} t^2}{\text{t}^2}$$

$$M = (.49664 \pm 65) + (14.869376 \pm 5) \text{t} + (2.13345 \pm 91) \times 10^{-3} \text{t}^2 - (1.449 \pm 26) \times 10^{-5} \text{t}^3$$

Standard error of one observation: $\sigma = \pm 6.0'$.

The following elements are based on 57 observations and are valid for the period October 22.0 through October 32.0, 1959.

$$T_o = 36868.0 \text{ MJD}$$

$$\omega = (211^\circ 776 \pm 57) - \frac{3877}{\text{t}} - \frac{1^\circ 0 \times 10^{-4} t^2}{\text{t}^2}$$

$$\Omega = (338^\circ 6517 \pm 60) - (3^\circ 1104 \pm 19) \text{t} - \frac{1^\circ 128 \times 10^{-3} t^2}{\text{t}^2}$$

$$i = 65^\circ 1347 \pm 56$$

$$e = (.055937 \pm 34) - (1.54 \pm 13) \times 10^{-4} \text{t} - \frac{1.29 \times 10^{-7} t^2}{\text{t}^2}$$

$$M = (.61952 \pm 17) + (14.963397 \pm 8) \text{t} + (2.3035 \pm 11) \times 10^{-3} \text{t}^2 - (7.65 \pm 53) \times 10^{-6} \text{t}^3$$

Standard error of one observation: $\sigma = \pm 6.0'$.

The following elements are based on 197 observations and are valid for the period November 1.0 through November 11.0, 1959.

$$T_o = 36878.0 \text{ MJD}$$

$$\omega = (207^{\circ}963 \pm 56) - \frac{42762}{t} - 3^{\circ}9 \times 10^{-4} t^2$$

$$\Omega = (307^{\circ}4603 \pm 59) - (3^{\circ}1283 \pm 21)t - \frac{1^{\circ}108}{t} \times 10^{-3} t^2$$

$$i = 65^{\circ}1523 \pm 67$$

$$e = (.053822 \pm 64) - \frac{1.852}{t} \times 10^{-4} - \frac{3.04}{t} \times 10^{-7} t^2$$

$$M = (.48506 \pm 18) + (15.010053 \pm 7)t + (2.2442 \pm 14) \times 10^{-3} t^2 - (1.023 \pm 49) \times 10^{-5} t^3$$

Standard error of one observation: $\sigma = \pm 5.4^{\circ}$.

The following elements are based on 58 observations and are valid for the period November 11.0 through November 21.0, 1959.

$$T_o = 36888.0 \text{ MJD}$$

$$\omega = \frac{203^{\circ}645}{t} - \frac{43538}{t^2} - 3^{\circ}9 \times 10^{-4} t^2$$

$$\Omega = (275^{\circ}977 \pm 40) - (3^{\circ}1542 \pm 61)t - \frac{1^{\circ}128}{t} \times 10^{-3} t^2$$

$$i = 65^{\circ}133 \pm 18$$

$$e = (.05208 \pm 14) - (2.25 \pm 49) \times 10^{-4} t - \frac{1.29}{t} \times 10^{-7} t^2$$

$$M = (.812729 \pm 82) + (15.056023 \pm 17)t + (2.3893 \pm 40) \times 10^{-3} t^2 + (7.74 \pm 60) \times 10^{-6} t^3$$

Standard error of one observation: $\sigma = \pm 6.0^{\circ}$.

The following elements are based on 162 observations and are valid for the period December 4.0 through December 14.0, 1959.

$$T_o = 36911.0 \text{ MJD}$$

$$\omega = (193^{\circ}262 \pm 73) - \frac{44522}{t} + \frac{1^{\circ}0}{t} \times 10^{-4} t^2$$

$$\Omega = (202^{\circ}8735 \pm 68) - \frac{3^{\circ}21496}{t} - \frac{1^{\circ}32}{t} \times 10^{-3} t^2$$

$$i = 65^{\circ}1172 \pm 23$$

$$e = (.047164 \pm 91) - \frac{2.144}{t} \times 10^{-4} t - \frac{6.9}{t} \times 10^{-7} t^2$$

$$M = (.42981 \pm 18) + (15.173966 \pm 4)t + (2.6969 \pm 15)t^2 + (3.50 \pm 28) \times 10^{-6} t^3$$

Standard error of one observation: $\sigma = \pm 6.0^{\circ}$.

The following elements are based on 93 observations and are valid for the period December 14.0 through December 20.0, 1959.

$$T_o = 36917.0 \text{ MJD}$$

$$\omega = (189^{\circ}56' \pm 45) - \frac{?44402 t}{1^{\circ}0 \times 10^{-4} t^2}$$

$$\Omega = (183^{\circ}514' \pm 21) - (3^{\circ}2308 \pm 84) t - \frac{1^{\circ}32 \times 10^{-3} t^2}{t^2}$$

$$i = 65^{\circ}145' \pm 31$$

$$e = (.04521 \pm 30) - \frac{2.23 \times 10^{-4}}{t} - \frac{6.9 \times 10^{-7}}{t^2}$$

$$M = (.5746 \pm 11) + (15.207128 \pm 16) t + (2.8051 \pm 61) \times 10^{-3} t^2 - (.19 \pm 21) \times 10^{-5} t^3$$

Standard error of one observation: $\sigma = \pm 1.2'$.

The following elements are based on 72 observations and are valid for the period December 20.0 through December 32.0, 1959.

$$T_o = 36926.0 \text{ MJD}$$

$$\omega = (186^{\circ}30' \pm 26) - \frac{?44222 t}{1^{\circ}0 \times 10^{-4} t^2}$$

$$\Omega = (154^{\circ}380' \pm 10) - (3^{\circ}2526 \pm 58) t - \frac{1^{\circ}32 \times 10^{-3} t^2}{t^2}$$

$$i = 65^{\circ}096' \pm 11$$

$$e = (.04442 \pm 12) - \frac{2.35 \times 10^{-4}}{t} - \frac{6.9 \times 10^{-7}}{t^2}$$

$$M = (.65715 \pm 80) + (15.255768 \pm 14) t + (2.7851 \pm 45) \times 10^{-3} t^2 + (1.39 \pm 11) \times 10^{-5} t^3$$

Standard error of one observation: $\sigma = \pm 9.0'$.

The following elements are based on 55 observations and are valid for the period January 10.0 to January 20.0, 1960.

$$T_o = 36948.0$$

$$\omega = (176.52 \pm 17) - \frac{4.3812 t}{1.0 \times 10^{-4} t^2}$$

$$\Omega = (82.243 \pm 25) - (3.3264 \pm 68) t - \frac{1.32 \times 10^{-3} t^2}{t^2}$$

$$i = 65.129' \pm 17$$

$$e = (.03898 \pm 18) + (.3 \times 10^{-4} \pm 1.4) t$$

$$M = (.74273 \pm 49) + (15.390666 \pm 31) t + (3.157 \pm 11) \times 10^{-3} t^2 + (4.19 \pm 42) \times 10^{-5} t^3$$

Standard error of one observation: $\sigma = \pm 12'$.

The following elements are based on 52 observations and are valid for the period January 24.0 to February 1.0, 1960.

$$T_o = 36961.0$$

$$\omega = (170.801 \pm 49) - \underline{.4736 t} - \underline{2.44 \times 10^{-4} t^2}$$

$$\Omega = (38.9156 \pm 60) - (3.3548 \pm 18) t - \underline{1.3 \times 10^{-3} t^2}$$

$$i = 65.1107 \pm 83$$

$$e = (.035284 \pm 46) - \underline{2.75 \times 10^{-4} t}$$

$$M = (.37713 \pm 15) + (15.478108 \pm 4) t + (3.548 \pm 13) \times 10^{-3} t^2 + (8.53 \pm 36) \times 10^{-6} t^3$$

Standard error of one observation: $\sigma = \pm 9.0'$.

The following elements are based on 35 observations and are valid for the period February 1.0 to February 7.0, 1960.

$$T_o = 36967.0$$

$$\omega = (167.87 \pm 13) - \underline{.4765 t} - \underline{2.44 \times 10^{-4} t^2}$$

$$\Omega = (18.826 \pm 17) - (3.3652 \pm 23) t - \underline{1.4 \times 10^{-3} t^2}$$

$$i = 65.181 \pm 14$$

$$e = (.03332 \pm 14) - (3.52 \pm 27) \times 10^{-4} t$$

$$M = (.37463 \pm 41) + (15.521335 \pm 9) t + (3.8050 \pm 16) \times 10^{-3} t^2 + (3.859 \pm 66) \times 10^{-5} t^3$$

Standard error of one observation: $\sigma = \pm 12'$.

The following elements are based on 21 observations and are valid for the period March 3.0 to March 11.0, 1960.

$$T_o = 37000.0$$

$$\omega = (151.34 \pm 22) - \underline{.431 t}$$

$$\Omega = (265.193 \pm 16) - (3.5180 \pm 75) t - \underline{5.89 \times 10^{-4} t^2}$$

$$i = 65.032 \pm 18$$

$$e = (.023637 \pm 80) - (3.48 \pm 63) \times 10^{-4} t$$

$$M = (.84112 \pm 64) + (15.521335 \pm 9) t + (3.8050 \pm 16) \times 10^{-3} t^2 + (3.859 \pm 66) \times 10^{-5} t^3$$

Standard error of one observation: $\sigma = \pm 15'$.

Date of demise: April 6, 1960

No elements are available for the period March 11 through April 6, because observations are lacking.

I. SAO smoothed elements. (An underline indicates an assumed value.)

The following elements are based on 237 observations and are valid for the period October 13.8 through October 31, 1959.

$$T_o = 36863.0 \text{ MJD}$$

$$\omega = (82^\circ 279 \pm 15) + (3^\circ 3830 \pm 25) t + \underline{4^\circ 0 \times 10^{-5} t^2} + \underline{1^\circ 257 \cos \omega}$$

$$\Omega = (113^\circ 2954 \pm 14) - (4^\circ 17619 \pm 26) t - \underline{2^\circ 9 \times 10^{-5} t^2} + \underline{.002 \cos \omega}$$

$$i = (50^\circ 3180 \pm 12) - \underline{.002 \sin \omega}$$

$$e = (.0369931 \pm 81) - (.03 \pm 16) \times 10^{-5} t + \underline{.000816 \sin \omega}$$

$$M = (.624292 \pm 42) + (14.211192 \pm 7) t + (3.3513 \pm 98) \times 10^{-5} t^2 - (3.12 \pm 14) \times 10^{-7} t^3 \\ - \underline{.00349 \cos \omega}$$

Standard error of one observation: $\sigma = \pm 10'$.

The following elements are based on 337 observations and are valid for the period November 1 through November 30, 1959.

$$T_o = 36888.0 \text{ MJD}$$

$$\omega = (167^\circ 1121 \pm 95) + (3^\circ 3891 \pm 11) t + \underline{4^\circ 0 \times 10^{-5} t^2} + \underline{1^\circ 257 \cos \omega}$$

$$\Omega = (8^\circ 8987 \pm 11) - (4^\circ 17600 \pm 13) t - \underline{2^\circ 9 \times 10^{-5} t^2} + \underline{.002 \cos \omega}$$

$$i = (50^\circ 30940 \pm 87) - \underline{.002 \sin \omega}$$

$$e = (.0369094 \pm 84) - (.30 \pm 11) \times 10^{-5} t + \underline{.000816 \sin \omega}$$

$$M = (.920755 \pm 27) + (14.212440 \pm 3) t + (2.0885 \pm 24) \times 10^{-5} t^2 + (4.41 \pm 34) \times 10^{-8} t^3 \\ - \underline{.00349 \cos \omega}$$

Standard error of one observation: $\sigma = \pm 10'$.

The following elements are based on 119 observations and are valid for the period December 1 through December 31, 1959.

$$T_o = 36918.0 \text{ MJD}$$

$$\omega = (268^\circ 685 \pm 37) + (3^\circ 3703 \pm 66) t + \underline{4^\circ 0 \times 10^{-5} t^2} + \underline{1^\circ 257 \cos \omega}$$

$$\Omega = (243^\circ 5939 \pm 39) - (4^\circ 17782 \pm 55) t - \underline{2^\circ 9 \times 10^{-5} t^2} + \underline{.002 \cos \omega}$$

$$i = (50^\circ 2971 \pm 37) - \underline{.002 \sin \omega}$$

$$e = (.037047 \pm 44) + (2.11 \pm 57) \times 10^{-5} t + \underline{.000816 \sin \omega}$$

$$M = (.31452 \pm 11) + (14.213782 \pm 20) t + (1.350 \pm 013) \times 10^{-5} t^2 - (2.62 \pm 14) \times 10^{-7} t^3 \\ - \underline{.00349 \sin \omega}$$

Standard error of one observation: $\sigma = \pm 8.0'$.

The following elements are based on 82 observations and are valid for the period January 1 through January 31, 1960.

$$\begin{aligned}
 T_o &= 36948.0 \text{ MJD} \\
 \omega &= (10^\circ 903 \pm 50) + (3^\circ 3922 \pm 56) t + \frac{\pm 40 \times 10^{-4} t^2}{\pm 1.257 \cos \omega} \\
 \Omega &= (118^\circ 2681 \pm 24) - (4^\circ 17767 \pm 34) t - \frac{\pm 29 \times 10^{-4} t^2}{\pm 20 \times 10^{-2} \cos \omega} \\
 i &= (50^\circ 3199 \pm 31) - \frac{\pm 20 \times 10^{-2} \sin \omega}{\pm 1.257 \cos \omega} \\
 e &= (.036659 \pm 17) + (.17 \pm 26) \times 10^{-5} t + \frac{\pm .816 \times 10^{-3} \sin \omega}{\pm 1.257 \cos \omega} \\
 M &= (.73715 \pm 14) + (14.214588 \pm 15) t + (.13634 \pm 56) \times 10^{-4} t^2 - (.416 \pm 53) \times 10^{-7} t^3 \\
 &\quad - \frac{\pm .349 \times 10^{-2} \cos \omega}{\pm 1.257 \cos \omega}
 \end{aligned}$$

Standard error of one observation: $\sigma = \pm 3.0'$.

The following elements are based on 116 observations and are valid for the period February 1 through February 29, 1960.

$$\begin{aligned}
 T_o &= 36979.0 \text{ MJD} \\
 \omega &= (116^\circ 242 \pm 49) + (3^\circ 3890 \pm 57) t + \frac{\pm 17 \times 10^{-4} t^2}{\pm 1.257 \cos \omega} \\
 \Omega &= (348^\circ 7400 \pm 25) - (4^\circ 1797 \pm 28) t - \frac{\pm 895 \times 10^{-5} t^2}{\pm 20 \times 10^{-2} \cos \omega} \\
 i &= (50^\circ 319 \pm 27) - \frac{\pm 20 \times 10^{-2} \sin \omega}{\pm 1.257 \cos \omega} \\
 e &= (.036834 \pm 23) + (.14 \pm 23) \times 10^{-5} t + \frac{\pm .816 \times 10^{-3} \sin \omega}{\pm 1.257 \cos \omega} \\
 M &= (-.40687 \pm 13) + (14.216028 \pm 15) t + (.12286 \pm 67) \times 10^{-4} t^2 - (.312 \pm 68) \times 10^{-6} t^3 \\
 &\quad - \frac{\pm .349 \times 10^{-2} \cos \omega}{\pm 1.257 \cos \omega}
 \end{aligned}$$

Standard error of one observation: $\sigma = \pm 3.5'$.

The following elements are based on 122 observations and are valid for the period March 1 through March 31, 1960.

$$\begin{aligned}
 T_o &= 37009.0 \text{ MJD} \\
 \omega &= (218^\circ 028 \pm 79) + (3^\circ 4068 \pm 72) t + \frac{\pm 40 \times 10^{-4} t^2}{\pm 1.257 \cos \omega} \\
 \Omega &= (223^\circ 3501 \pm 56) - (4^\circ 18208 \pm 46) t - \frac{\pm 29 \times 10^{-4} t^2}{\pm 20 \times 10^{-2} \cos \omega} \\
 i &= (50^\circ 2891 \pm 31) - \frac{\pm 20 \times 10^{-2} \sin \omega}{\pm 1.257 \cos \omega} \\
 e &= (.036695 \pm 46) - (.49 \pm 40) \times 10^{-5} t + \frac{\pm .816 \times 10^{-3} \sin \omega}{\pm 1.257 \cos \omega} \\
 M &= (-.88910 \pm 23) + (14.216786 \pm 22) t + (.2554 \pm 11) \times 10^{-4} t^2 + (.1433 \pm 86) \times 10^{-6} t^3 \\
 &\quad - \frac{\pm .349 \times 10^{-2} \cos \omega}{\pm 1.257 \cos \omega}
 \end{aligned}$$

Standard error of one observation: $\sigma = \pm 3.0'$.

The following elements are based on 189 observations and are valid for the period April 1 through April 30, 1960.

$$T_o = 37040.0 \text{ MJD}$$

$$\omega = (323^{\circ}446 \pm 35) + (3^{\circ}4341 \pm 65)t + .57 \times 10^{-4}t^2 + 1^{\circ}257 \cos \omega$$

$$\Omega = (93^{\circ}7993 \pm 19) - (4^{\circ}17997 \pm 38) - .51 \times 10^{-4}t^2 + .20 \times 10^{-2} \cos \omega$$

$$i = (50^{\circ}3172 \pm 17) - .20 \times 10^{-2} \sin \omega$$

$$e = (.036731 \pm 20) - (.29 \pm 33) \times 10^{-5}t + .816 \times 10^{-3} \sin \omega$$

$$M = (.63890 \pm 10) + (14.218590 \pm 19)t + (.32042 \pm 91) \times 10^{-4}t^2 + (.1924 \pm 83) \times 10^{-6}t^3 \\ - .349 \times 10^{-2} \cos \omega$$

Standard error of one observation : $\sigma = \pm 3.0'$.

Estimated date of demise : 1990

II. SAO mean elements

(NJD)	ω	Ω	i	e	M	n	n'	a	N	D	C
36907.0	230.83 3	289.539 7	50.298 5	.03621 6	.9671 1	14.213225 3	.244E-4 9	7.198756	65	8	.84
36911.0	244.63 4	272.81 1	50.282 8	.03603 7	.8201 1	14.213369 3	.18E-4 1	7.198706	42	8	.78
36915.0	258.51 8	256.10 5	50.29 1	.0360 2	.6735 2	14.213468 6	.12E-4 2	7.198673	22	8	.59
36919.0	275. 4	239.5 2	50.3 2	.035 4	.52 4	14.21358 6	.1E-4 2	7.198635	27	8	.397
36923.0	286.9 6	222.72 1	50.29 1	.0359 3	.380 2	14.213697 4	.8E-5 3	7.198596	28	8	.80
36927.0	302. 6	206.0 2	50.3 2	.036 3	.23 2	14.2137 1	-.4E-4 7	7.19855	15	8	.837
36937.0	-24.6 3	164.187 8	50.291 7	.0357 2	.3738 7	14.214164 2	.18E-4 1	7.198438	19	8	.58
36941.0	-10.3 4	147.47 1	50.284 9	.0354 3	.2292 3	14.214351 2	.174E-4 7	7.198374	25	8	.57
36945.0	2. 1	130.80 6	50.30 3	.037 2	.089 3	14.214522 8	.11E-4 5	7.198318	17	8	.40
36949.0	16.2 7	114.1 1	50.32 7	.0369 3	.947 2	14.21470 4	-.02E-4 1	7.198258	15	8	.650
36953.0	31. 2	97.5 3	50.3 2	.0368 6	.803 5	14.21479 6	.1E-4 3	7.198224	19	8	.40
36957.0	42.4 2	80.66 3	50.32 2	.03726 6	.6669 6	14.214951 7	.12E-4 2	7.198175	21	8	1.88
36961.0	55.67 8	63.951 6	50.321 5	.03736 3	.5272 2	14.215089 2	.165E-4 9	7.198128	31	8	.79
36965.0	68.92 8	47.240 3	50.319 3	.03749 2	.3879 2	14.215257 2	.21E-4 1	7.198071	35	8	1.00
36969.0	82.18 9	30.529 5	50.314 6	.03755 3	.2494 2	14.215465 2	.258E-4 9	7.198001	26	8	1.09
36973.0	95.5 1	13.83 2	50.31 2	.03764 7	.1116 4	14.215649 8	.20E-4 2	7.197938	22	8	.77
36977.0	108.8 1	-2.92 1	50.29 1	.03766 7	.9744 3	14.215772 4	.17E-4 2	7.197895	25	8	.62
36981.0	122.0 1	-19.622 9	50.299 8	.03767 6	.8379 3	14.215880 3	.152E-4 8	7.197859	40	8	.54
36985.0	135.8 1	-36.29 1	50.291 3	.03726 4	.7006 3	14.215989 5	.13E-4 1	7.197822	50	8	.54
36989.0	148.96 8	-53.09 1	50.335 9	.03699 4	.5653 2	14.216068 3	.17E-4 1	7.197799	34	8	.56
36993.0	162.4 1	-69.74 1	50.28 2	.0369 1	.4297 3	14.216150 4	.22E-4 2	7.197767	18	8	1.33
36997.0	175.90 6	-86.473 4	50.290 5	.03683 7	.2946 2	14.216244 4	.22E-4 1	7.197736	30	8	1.29
37001.0	189.49 5	-103.203 5	50.294 5	.03665 7	.1599 1	14.216356 4	.15E-4 2	7.197698	35	8	1.40
37005.0	203.1 1	-119.90 2	50.29 1	.03674 3	.0258 3	14.21653 1	.33E-4 5	7.197641	18	8	2.23
37009.0	216.98 7	-136.633 9	50.308 9	.03617 7	.8920 2	14.216662 5	.26E-4 2	7.197596	20	8	.82
37013.0	227. 1	-153.37 1	50.292 9	.03579 8	.770 4	14.216835 3	.27E-4 1	7.197536	38	8	.67
37017.0	248. 1	-170.06 2	50.32 1	.0364 2	.618 4	14.217031 2	.255E-4 8	7.197472	50	8	.49
37021.0	261. 1	-186.81 3	50.31 2	.0362 3	.489 4	14.217260 3	.35E-4 2	7.197395	39	8	.65
37025.0	272.49 6	-203.494 9	50.315 8	.03583 7	.3649 2	14.217497 3	.46E-4 2	7.197315	50	8	1.12
37029.0	286.58 6	-220.19 1	50.324 9	.03613 9	.2353 2	14.217773 5	.18E-4 2	7.197222	51	8	1.03
37033.0	300.12 4	-236.942 2	50.320 2	.03604 4	.1083 1	14.217997 2	.233E-4 7	7.197146	59	8	.79
37037.0	313.99 4	-253.662 2	50.316 2	.03611 3	.9811 1	14.218245 2	.309E-4 7	7.197063	68	8	.90
37041.0	327.95 7	-270.382 3	50.314 3	.03628 2	.8545 2	14.218543 2	.33E-4 1	7.196962	74	8	.89
37045.0	341.79 6	-287.094 4	50.320 3	.03646 1	.7294 2	14.218859 2	.336E-4 8	7.196856	62	8	.62
37049.0	355.6 1	-303.815 8	50.320 4	.03664 3	.6055 3	14.219193 4	.37E-4 1	7.196743	37	8	.80
37053.0	368.9 5	-320.51 5	50.29 3	.0369 1	.484 2	14.21953 1	.32E-4 5	7.196628	22	8	3.12

TABLE 3

RELATIVE POSITIONS OF THE SUN AND THE PERIGEE OF SATELLITE 1959 41

T (MJD)	ω	Ω	Ψ	$\Delta\alpha$	φ	C (km)
36857.	62.47	138.35	52.55	348.95	43.05	10.0
36861.	75.85	121.64	59.53	346.07	48.24	12.0
36865.	88.96	104.94	62.96	345.75	50.30	12.7
36869.	102.29	88.24	62.84	345.60	48.73	12.1
36873.	115.67	71.54	60.07	343.02	43.92	10.3
36877.	129.01	54.83	56.40	337.16	36.74	7.7
36881.	142.36	38.12	54.04	328.52	28.04	4.7
36885.	155.77	21.43	54.87	317.97	18.41	2.1
36889.	169.31	4.72	59.42	306.30	-8.24	0.4
36893.	182.94	348.01	66.57	294.15	-2.18	0.0
36897.	196.48	331.31	74.43	282.11	-12.59	1.0
36901.	210.09	314.61	81.21	270.82	-22.74	3.2
36905.	223.96	297.91	85.55	261.04	-32.26	6.1
36909.	237.73	281.18	87.00	253.73	-40.59	9.1
36913.	251.58	264.47	86.18	249.77	-46.89	11.4
36917.	265.45	247.76	84.72	249.09	-50.09	12.6
36921.	279.33	231.05	84.71	249.48	-49.40	12.4
36925.	293.20	214.35	87.94	247.77	-45.01	10.7
36929.	307.04	197.64	95.21	242.51	-37.90	8.1
36933.	320.84	180.93	106.24	234.13	-29.08	5.1
36937.	334.58	164.22	120.11	223.60	-19.29	2.3
36941.	348.26	147.51	135.74	211.83	-9.00	0.5
36945.	1.88	130.80	152.13	199.52	1.45	0.0
36949.	15.42	114.08	168.34	187.28	11.81	0.9
36953.	28.90	97.37	175.75	175.72	21.83	3.0
36957.	42.31	80.66	162.48	165.53	31.20	5.8
36961.	55.67	63.95	151.46	157.56	39.46	8.7
36965.	69.00	47.24	143.58	152.63	45.92	11.1
36969.	82.29	30.52	139.15	150.90	49.69	12.5
36973.	95.58	13.81	137.83	150.83	49.98	12.6
36977.	108.87	357.09	138.38	149.62	46.73	11.4
36981.	122.19	340.38	138.60	145.43	40.63	9.1
36985.	135.55	323.67	135.98	138.19	32.61	6.2
36989.	148.95	306.95	129.18	128.71	23.38	3.4
36993.	162.42	290.23	118.80	117.80	13.44	1.2
36997.	175.96	273.52	106.44	106.20	3.11	0.1
37001.	189.57	256.80	93.81	94.50	-7.35	0.4
37005.	203.25	240.08	82.59	83.29	-17.68	2.0
37009.	216.99	223.37	74.34	73.26	-27.58	4.6
37013.	230.79	206.65	70.21	65.25	-36.60	7.6
37017.	244.63	189.93	70.10	60.25	-44.05	10.4
37021.	258.50	173.21	72.46	58.82	-48.94	12.2
37025.	272.39	156.49	74.91	59.86	-50.25	12.7
37029.	286.28	139.77	75.24	60.32	-47.62	11.7
37033.	300.14	123.05	71.99	57.65	-41.72	9.5
37037.	313.97	106.33	64.79	51.47	-33.63	6.6
37041.	327.76	89.61	54.14	42.62	-24.24	3.6
37045.	341.48	72.88	41.00	32.04	-14.15	1.3
37049.	355.14	56.16	26.46	20.53	-3.74	0.1
37053.	8.73	39.44	11.52	8.73	6.71	0.3

ON THE ORBITAL ELEMENTS OF SATELLITE 1959 61

by

Yoshihide Kozai¹

The orbital elements

During the period October 13 to December 3, 1959, Satellite 1959 61 (Explorer VII) was transmitting on 108 Mc/s. During this time a number of observations with good distribution both in time and in true anomaly were made at Minitrack Stations. From the Minitrack data, combined with Baker-Nunn photographic (field-reduced) and Moonwatch observations, orbital elements have been determined by the SAO Differential Orbit Improvement Program (DOI) of G. Veis and C. Moore.

Observations made on October 13 (the launching date) were rejected because of their rather poor accuracy. Weights inversely proportional to the squares of 660, 184, and 3300 were assigned, respectively, to the Minitrack, Baker-Nunn, and Moonwatch observations.

The orbital elements, tabulated in Table 1 (p. 22), were derived at two-day intervals from October 16.0 (U. T.) by use of sets of observations covering periods of four days, two days on each side of the epoch. These are mean elements in the sense that the short-periodic and a part of the long-periodic perturbations have been subtracted to derive these elements. Therefore, to invert to osculating values of the eccentricity (e), the inclination (i), the argument of perigee (ω), and the longitude of the ascending

node (Ω), we must add the quantities $d\epsilon_s$, d_i , $d\omega_s - \frac{3}{8} \frac{A_2}{p^2} \sin^2 i \sin 2\omega$, and $d\Omega_s$. The quantities $d\epsilon_s$, d_i , $d\omega_s$, $d\Omega_s$, and dM_s have the same expressions as those given in an earlier paper (Kozai, 1959c).

The mean anomaly and the anomalistic mean motion given in Table 1 are defined as follows:

$$M_{(\text{oscillating})} = M + nt + dM_s + 1.06 \cos \omega + \frac{1}{2} n t^2.$$

$1.06 \cos \omega$ is an assumed term due to odd-harmonics of the earth's gravitational potential, and the term $\frac{1}{2} n t^2$ results from air-drag. Therefore the variations in the values of the mean motions from epoch to epoch are principally due to air-drag, and not to perturbations by the earth's potential.

The reference plane is defined as the true equator of date. The origin of right ascension is a line shifted from the mean equinox of date by an amount equal to the precession in right ascension between 1950.0 and the date.

The long-period terms

To derive the amplitudes of long-periodic terms with argument ω and the secular motions of the perigee and the node, the same technique as in a previous paper (Kozai, 1959a) is adopted.

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TABLE 1

SAO MEAN ELEMENTS OF SATELLITE 1959 1.1

October 16 through December 3, 1959

T (MJD)	ω	Ω	i	e	M	n	$\Delta n \times 10^5$	$\Delta M \times 10^4$
36857•	62•474 53	138•351 6	50•316 5	•03770 2	•35843 15	14•210786 8	-139	155
36859•	69•244 42	129•993 4	50•313 4	•03773 2	•77980 12	14•210925 5	-125	125
36861•	75•852 36	121•643 3	50•319 3	•03780 2	•20184 10	14•211079 4	-110	102
36863•	82•364 38	113•290 2	50•320 2	•03782 1	•62448 11	14•211213 4	-96	85
36865•	88•964 31	104•943 3	50•321 3	•03778 2	•04708 9	14•211340 4	-84	67
36867•	95•658 51	96•584 4	50•319 5	•03775 3	•46972 14	14•211467 7	-71	50
36869•	102•286 60	88•235 5	50•318 5	•03773 4	•89288 16	14•211566 9	-61	38
36871•	108•913 36	79•887 4	50•317 3	•03770 3	•31604 10	14•211665 8	-51	26
36873•	115•673 40	71•537 5	50•312 3	•03767 4	•73915 11	14•211796 4	-38	14
36875•	122•381 32	63•185 4	50•310 2	•03755 3	•16271 9	14•211929 3	-25	6
36877•	129•012 28	54•829 4	50•311 3	•03752 2	•58677 8	14•212028 4	-15	3
36879•	135•707 31	46•479 4	50•312 3	•03750 2	•01083 9	14•212105 5	-7	0
36881•	142•365 29	38•125 3	50•313 2	•03743 2	•43518 8	14•212176 3	0	0
36883•	149•061 26	29•777 2	50•310 2	•03736 1	•85960 8	14•212245 2	7	1
36885•	155•773 23	21•427 2	50•311 2	•03730 1	•28414 7	14•212321 3	15	3
36887•	162•570 22	13•077 3	50•309 2	•03722 2	•70868 6	14•212421 3	25	4
36889•	169•309 20	4•722 3	50•311 2	•03710 2	•13361 6	14•212492 4	32	10
36891•	176•089 56	-3•641 6	50•306 4	•03697 4	•55864 15	14•212575 5	40	17
36893•	182•937 34	-11•995 4	50•302 3	•03683 3	•98367 9	14•212656 4	48	24
36895•	189•743 29	-20•347 3	50•304 3	•03675 3	•40903 8	14•212744 4	57	34
36897•	196•484 28	-28•687 3	50•311 3	•03665 4	•83476 8	14•212813 5	64	48
36899•	203•264 21	-37•037 3	50•314 3	•03652 3	•26058 6	14•212909 4	73	62
36901•	210•088 25	-45•393 4	50•312 4	•03645 4	•88657 7	14•213028 4	85	79
36903•	217•002 31	-53•744 5	50•306 3	•03634 4	•11256 9	14•213121 6	95	95
36905•	223•958 68	-62•094 8	50•302 5	•03627 6	•53862 21	14•213212 7	104	112

For this series of observations, November 9.0, 1959 (U.T.) is the mean epoch. From the difference Δn between the mean motion of each and the time 14.212176 of the mean epoch, the effect of air-drag on the eccentricity Δe is computed as:

$$\Delta e = -\frac{2}{3} \frac{1-e}{n} \Delta n = -.0452 \Delta n,$$

on the assumption that air-drag does not have any effect on perigee height.

Air-drag effects on the perigee and the node are given by the expressions:

$$\Delta \omega = \frac{A_2}{3p^2} \frac{4-5 \sin^2 i}{2} \frac{7-e}{1+e} \Delta M = .536 \Delta M,$$

$$\Delta \Omega = -\frac{A_2}{3p^2} \cos i \frac{7-e}{1+e} \Delta M = -.658 \Delta M,$$

where

$$\Delta M = M - .43518 - 14.212176 t,$$

and time is measured from the mean epoch in days.

After these values are subtracted from the eccentricity, the argument of perigee, and the node, the remaining variations of the orbital elements may be due to gravitational forces resulting from the oblateness of the earth, to the sun and the moon, and to the solar radiation pressure (Musen, Bryant and Bailie, 1960).

Luni-solar perturbations (Kozai, 1959b), derived by the writer's IBM-704 program, are shown in Table 2 for each epoch and each element. Besides these, the long-periodic terms with argument 2 ω (Brouwer, 1959; Garfinkel, 1959; Kozai, 1959c) must be subtracted. If the value of A_4/a_e^4 is 9.16×10^{-6} , the amplitudes of these terms in the eccentricity and the argument of perigee are, respectively, -1.3×10^{-5} and $.020$. The amplitudes in both the inclination and the node are negligibly small. No correction has been made for the effect of the solar radiation pressure.

TABLE 2

LUNI-SOLAR PERTURBATIONS

MJD	$de \times 10^6$	$di \times 10^4$	$d\omega \times 10^4$	$d\Omega \times 10^4$
36857.	16	-13°	32°	10°
36859.	17	-18	52	3
36861.	17	-17	56	-4
36863.	16	-11	43	-7
36865.	17	-7	24	-7
36867.	18	-8	16	-8
36869.	19	-8	21	-14
36871.	19	-5	30	-20
36873.	17	3	27	-21
36875.	15	9	8	-17

36877.	14	11	-21	-12
36879.	14	10	-46	-12
36881.	14	11	-54	-13
36883.	13	15	-43	-12
36885.	12	17	-27	- 6
36887.	9	16	-23	1
36889.	7	12	-41	2
36891.	7	10	-70	0
36893.	7	11	-90	- 2
36895.	8	13	-87	1
36897.	9	10	-70	3
36899.	8	6	-62	2
36901.	6	5	-75	- 3
36903.	5	8	-107	- 7
36905.	6	11	-142	- 5

Results

After all corrections are made, the four elements, e , i , ω , and Ω may be expressed as :

$$e = e_0 + \delta e \sin \omega,$$

$$i = i_0 + \delta i \sin \omega,$$

$$\omega = \omega_0 + \dot{\omega} t + \delta \omega \cos \omega,$$

$$\Omega = \Omega_0 + \dot{\Omega} t + \delta \Omega \cos \omega.$$

Except for δi and $\delta \Omega$, which were assumed to be zero and ± 0.0016 respectively, all constants are solved for by the method of least squares by assigning equal weights to all four-day orbits.

Unfortunately, because the observations used in this paper do not cover the whole period of the term with argument ω , the correlation between coefficients of unknown constants in the equations of conditions is rather strong. The results are tabulated in Table 3.

TABLE 3

ORBITAL RESULTS BY THE METHOD OF LEAST SQUARES
(Mean epoch, November 9.0, 1959 U.T.)

	Mean value	Amplitude	Secular motion	Probable error of one equation
Eccentricity	.036919 \pm 7	.000815 \pm 8		\pm .000027
Inclination	50°31'23" \pm 9			\pm °0045
Perigee	143°26'8" \pm 17	1°18'3" \pm 37	3°39'17" \pm 12	\pm °042
Node	38°13'04" \pm 15		-4°17'59" \pm 10	\pm °0036

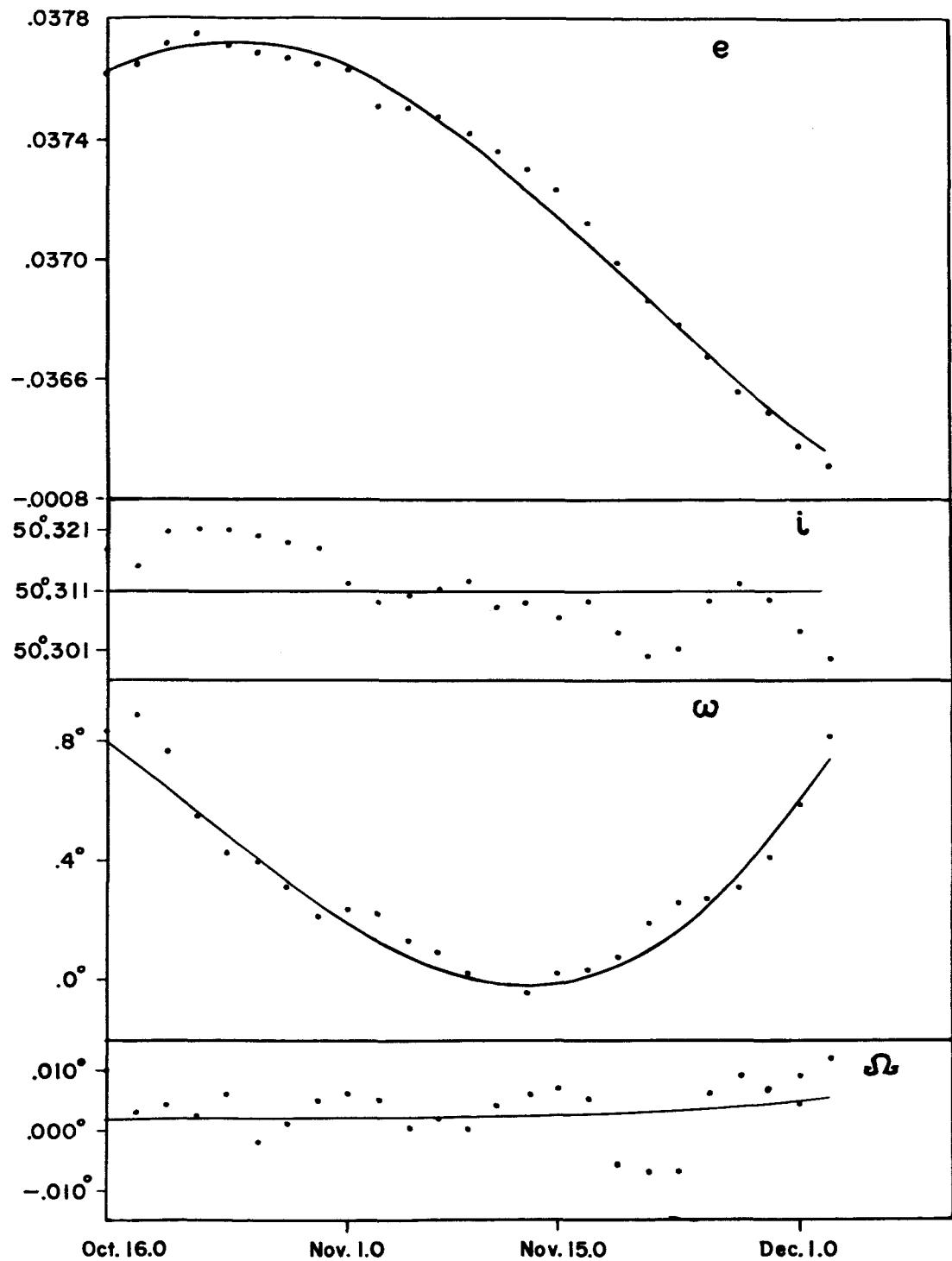


FIGURE 1.--Dots represent observed values of the orbital elements (corrected for luni-solar, air-drag, and long-period perturbations of argument 2ω). For ω and Ω , secular terms are taken into account with preliminary values.

In Figure 1, observed values are plotted for each element with the best-fit curves. However, instead of ω and Ω themselves, the figure plots the quantities, $\omega - 142^\circ 341 - 3^\circ 365 t$ and $\Omega - 38^\circ 126 + 4^\circ 176 t$.

Conclusions

Predicted amplitudes of the eccentricity and the argument of perigee are .000693 and $1^\circ 076$, if 2.2×10^{-6} is assigned to the value of the third harmonic of the earth's potential (Kozai, 1959a). Differences between the observed and predicted amplitudes show that not only the third harmonic but also the fifth harmonic of the earth's potential must have significant value (O'Keefe, Eckels, and Squires, 1959).

Moreover, some systematic deviations from the best-fit curves are evident in Figure 1. These deviations may disappear if some longitudinal harmonics of the earth and the solar radiation effects are added.

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NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory. First issued to ensure the immediate dissemination of data for satellite tracking, the Reports have continued to provide a rapid distribution of catalogues of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals.

Edited and produced under the supervision of Mrs. L. G. Boyd and Mr. E. N. Hayes, the Reports are indexed by the Science and Technology Division of the Library of Congress, and are regularly distributed to all institutions participating in the U. S. space research program and to individual scientists who request them from the Administrative Officer, Technical Information, Smithsonian Astrophysical Observatory, Cambridge 38, Massachusetts.

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